

*FALSIFICATION OF MATCHING THEORY:  
CHANGES IN THE ASYMPTOTE OF HERRNSTEIN'S  
HYPERBOLA AS A FUNCTION OF  
WATER DEPRIVATION*

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Five rats pressed levers on variable-interval schedules of water reinforcement at various levels of water deprivation. In one phase of the experiment, three deprivation conditions that replicated conditions in Heyman and Monaghan (1987) were arranged, along with three less extreme deprivation conditions. In a second phase, water deprivation was arranged so that subjects were exposed to a greater range of access to water per day. Herrnstein's hyperbola described the rats' response-rate data well. The  $y$  asymptote,  $k$ , of the hyperbola appeared roughly constant over the conditions that replicated those of Heyman and Monaghan, but decreased markedly when less extreme deprivation conditions were included. In addition,  $k$  varied systematically when the second method of arranging deprivation was used. These results falsify a strong form of matching theory and confirm predictions made by linear system theory.

*Key words:* Herrnstein's hyperbola, matching theory, linear system theory, reinforcer magnitude, water deprivation, lever press, rats

Herrnstein's (1970) hyperbola,

$$R = \frac{kr}{r + r_e}, \quad (1)$$

describes the relationship between response rate,  $R$ , and reinforcement rate,  $r$ , on single-alternative schedules of reinforcement. The parameter  $k$  is the  $y$  asymptote of the hyperbola and represents the total amount of behavior that can be exhibited in a given environment. The parameter  $r_e$  represents the rate of reinforcement obtained for responses other than the instrumental response. Nearly 30 years of research on schedule performance has left little doubt that the hyperbolic form of Equation 1 accurately describes the relationship between response and reinforcement rates on single-alternative schedules (McDowell, 1988).

Herrnstein (1970) obtained Equation 1 from his original matching equation by conceptualizing single-alternative schedules as two-alternative concurrent schedules with the instrumental response as one alternative and

the aggregate of all other behavior in the operant chamber as the other alternative. He then assumed that the sum of the response rates on the two alternatives, which he designated  $k$ , was constant across changes in reinforcer properties. This assumption permitted Herrnstein to obtain Equation 1 from the matching equation. Without this assumption, it is not possible to isolate the absolute rate of instrumental responding,  $R$ , on one side of the equation, and hence an expression relating absolute response rate and absolute reinforcement rate cannot be obtained. In a later paper, Herrnstein (1974) noted that the constant  $k$  assumption provides a means of falsifying Equation 1 empirically. If  $k$  is found to vary with changes in reinforcer properties, then Equation 1 cannot be obtained algebraically from the matching equation, and Herrnstein's (1970) conceptualization of single-alternative responding as choice between concurrently available alternatives is called into question (McDowell, 1986).

The constant  $k$  assumption has not fared as well as the hyperbolic form of Equation 1 in empirical studies. In deVilliers' (1977) early review of incidental findings on  $k$ , he reported mixed outcomes, some supporting the constancy of  $k$  and others not. Subsequent reviews (McDowell, 1980; Warren-Boulton, Silberberg, Gray, & Ollom, 1985; Williams, 1988) reached similar conclusions. For ex-

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ample, several experiments have shown that  $k$  does vary with reinforcer properties such as concentration of sucrose in water (Bradshaw, Szabadi, & Bevan, 1978; Heyman & Monaghan, 1994, first presentation), volume of sucrose (Schrier, 1965, reanalyzed by deVilliers, 1977), type of food (Keesey & Kling, 1961, reanalyzed by deVilliers, 1977), body weight (Snyderman, 1983), shock voltage reduction (Campbell & Kraeling, 1953, reanalyzed by deVilliers, 1977), and intensity of brain stimulation (Keesey, 1962, 1964, reanalyzed by deVilliers, 1977), whereas other experiments have shown no systematic change in  $k$  when the volume of the reinforcer (Bradshaw, Ruddle, & Szabadi, 1981), duration of access to the reinforcer (Kraeling, 1961, reanalyzed by deVilliers, 1977), use of glucose versus sucrose as the reinforcer (Guttman, 1954, reanalyzed by deVilliers, 1977), hours of food deprivation (Logan, 1960, reanalyzed by deVilliers, 1977), or concentration of sucrose in water (Heyman & Monaghan, 1994, second presentation) was manipulated.

McDowell and Wood (1984, 1985) conducted the most extensive and systematic experiments directly investigating the constancy of  $k$ . They found that  $k$  varied as a function of reinforcer magnitude. In one study, for example, McDowell and Wood (1984) reported that the median  $k$  across human subjects in their experiment varied 115% from the lowest to the highest reinforcer magnitude (cents per reinforcement). McDowell and Wood (1984, 1985) also noted that a mathematical account known as linear system theory predicted the form of the variation in  $k$  they observed. This theory was developed by McDowell and his colleagues (McDowell, 1987; McDowell, Bass, & Kessel, 1983, 1993; McDowell & Kessel, 1979), who showed that the mathematical theory of linear systems (Aseltine, 1958; Brown, 1961) can be used to generate an equation that relates absolute response rate,  $R_{\text{out}}$ , to absolute reinforcement rate,  $R_{\text{in}}$ :

$$R_{\text{out}} = \left\{ \ln \left[ 1 + \frac{P_B}{\gamma P_R} (e^{1/R_{\text{in}} + w} - 1) \right] - w^* \right\}^{-1}. \quad (2)$$

In this equation,  $P_B$  represents response aver-siveness or cost,  $P_R$  represents reinforcer value,  $w$  and  $w^*$  represent the durations of re-

inforcer and response events, and  $\gamma$  is a scalar constant that represents properties of the organism. McDowell (1980) showed that the form of Equation 2 is indistinguishable from the hyperbolic form of Equation 1 at ordinary rates of responding and reinforcement, and that the linear system theory permits Herrnstein's  $k$  to vary with reinforcer value, although it specifies circumstances under which the change in  $k$  may be small and therefore difficult to detect. The relationship between Equation 2 and Herrnstein's  $k$  is explained in Appendix A.

McDowell and Wood (1984, 1985) argued that existing data on the constancy of  $k$  were consistent with the linear system theory. According to the theory, there are circumstances under which  $k$  may appear to be invariant, but given appropriate experimental conditions (such as the ones they arranged)  $k$  can be shown to vary with reinforcer magnitude, in direct violation of the strongest form of matching theory. Soon after McDowell and Wood's papers appeared, their data and conclusions were challenged by Heyman and Monaghan (1987), who reported the results of experiments that seemed to show an invariant  $k$ . They studied rats' lever pressing on variable-interval (VI) schedules of water reinforcement at three levels of water deprivation, namely, 6, 23.5, and 47.5 hr of deprivation. The different levels of water deprivation were presumed to produce different reinforcer values. Heyman and Monaghan obtained a  $k$  for each condition and found that it was roughly constant across the three deprivation conditions. In addition, they dismissed McDowell and Wood's findings by arguing that Equation 1 did not describe some of their response-rate versus reinforcement-rate data well, and hence that their findings with respect to  $k$  should be disregarded. Given their own results, and having dismissed the most extensive and systematic contradictory findings, Heyman and Monaghan concluded that  $k$  was indeed constant across changes in reinforcer properties other than rate, just as matching theory requires.

The principal purpose of the experiment reported in this article was to investigate the possibility that Heyman and Monaghan's (1987) results were due to their use of a range of reinforcer magnitudes (i.e., deprivation levels) that was too small to produce

detectable changes in  $k$ . In the first phase of the experiment, Heyman and Monaghan's 6-, 23.5-, and 47.5-hr deprivation conditions were replicated using rats as subjects, and 0-, 2-, and 4-hr deprivation conditions were added. In this phase, Heyman and Monaghan's method of arranging deprivation was used. This method allows at least 30 min of access to water immediately after each experimental session and then 5 min of access to water at the nominal deprivation time. For example, in the 6-hr deprivation condition, water was available for 30 min immediately following each session, and was again available 6 hr before the next session for a period of 5 min. Notice that this method produces fairly severe water deprivation regardless of the nominal value of the condition. Even in the 0-, 2-, 4-, and 6-hr deprivation conditions, for example, rats had access to water for only 35 min each day: 30 min immediately following each session and 5 min 0, 2, 4, or 6 hr before each session. In the second phase of the present experiment deprivation was arranged according to the actual nominal values of 4, 6, 12, and 18 hr of deprivation. In these conditions, water was available in the home cage except for the 4, 6, 12, or 18 hr preceding each experimental session. These deprivation conditions were much milder than those used in the first phase of the experiment and presumably yielded smaller reinforcer magnitudes. In addition to the conditions in these two phases of the experiment, two additional conditions were arranged to test for the possible confounding effect of immediate post-session watering.

## METHOD

### *Subjects*

Five experimentally naive male Long Evans hooded rats served as subjects. Animals were approximately 140 days old at the beginning of the experiment and were housed in individual cages. Food (Purina Rat Chow) was freely available in the home cages at all times. Water was available according to the deprivation schedule described below. The colony room was under a light-dark cycle of 12 hr.

### *Apparatus*

Three standard operant chambers were used for all experiments (Med Associates,

two-lever operant conditioning chamber, Model ENV-001). The interior dimensions of each chamber were 28 cm long by 21 cm wide by 21 cm high. Two response levers were located on the front panel 8.6 cm above the grid floor and 6 cm to the left and right of center. Only the right lever was operative. A minimum force of 0.15 N was required to register a response. The force requirement could be increased by adding 10-, 20-, 30-, or 50-g weights to a metal hook protruding from the back side of the lever. A 28-VDC light covered by a 2.5-cm frosted lens was mounted above each lever, 14 cm above the floor. Each chamber also contained a 28-VDC houselight and a Sonalert® tone generator. A hole (5.1 cm) in the front panel centered 5.4 cm above the grid floor allowed access to a recessed pan (3.0 cm diameter) in which water was delivered. The liquid dispenser was located above the pan, and delivered 0.025-ml droplets of water. Each chamber was equipped with a white noise generator to mask extraneous sounds, and was contained within a sound-attenuating cubicle. Ventilation fans were located on the back sides of the cubicles. Events within the chamber were controlled and data were recorded by a computer operating under MED-PC® software.

### *Procedure*

*Pretraining.* Rats' lever pressing was hand shaped during two to four 70-min sessions. After obtaining 70 reinforcers, rats were exposed to a variable-ratio (VR) 5 schedule until steady responding was established. For the next 12 sessions, rats continued to work on the VR 5, but the force required to register a response was gradually increased. The force requirement was increased from 10 g to 20 g to 30 g, and then to 50 g. Approximately three sessions were required at each weight level. The force requirement remained at 50 g throughout Phases 1 and 2.

*Phase 1 deprivation.* Rats were exposed to a series of VI schedules at six levels of water deprivation. The nominal values of the deprivation conditions were 0, 2, 4, 6, 23.5, and 47 hr of deprivation. In the 47-hr condition, rats received 60 min of free access to water in their home cages immediately following each session. In the 23.5-hr condition, rats received 30 min of free access to water in their home cages immediately following each ses-

sion. In the remaining conditions, rats received 30 min of free access to water in their home cages immediately following each session and then 5 min of free access to water in their home cages 0, 2, 4, or 6 hr before the start of the next session. Following each session, the rats' water-drinking behavior was observed in their home cages to test for satiation. Sessions were conducted on alternate days in the 47-hr condition, except as noted below, and daily in the other conditions. All sessions were conducted at the same time each day. The order of presentation of deprivation conditions is listed in Table 1 and was random without replacement for each rat; that is, the first deprivation condition was selected at random, then the second deprivation condition was selected at random from the remaining five conditions, and so on. All rats lost weight rapidly in the 47-hr condition. Because of severe weight loss, R1, R2, and R14 were removed periodically from the deprivation regime and given free access to water. The regime was suspended from two to seven times during the 47-hr condition for these rats, and remained suspended for 1 to 4 days on each occasion. Rats did not participate in experimental sessions during periods of free access to water.

*Phase 2 deprivation.* The total duration of access to water per day varied little in the Phase 1 deprivation conditions, and the conditions were fairly severe. In Phase 2, four deprivation conditions were chosen to produce greater variability in the total duration of access to water per day and less severe water deprivation. Specifically, 4-, 6-, 12-, and 18-hr deprivation conditions were arranged such that the nominal value of each condition specified the number of hours before each experimental session during which water was unavailable in the home cages. Water was available in the home cages at all other times. After each session, the rats' water-drinking behavior was observed in their home cages to test for satiation. Sessions were conducted daily and at the same time each day. The order of presentation of deprivation conditions is listed in Table 1 and was random without replacement for each rat.

*VI series and discriminative stimuli.* Experimental sessions for both Phase 1 and Phase 2 consisted of five VI components separated by blackouts. During sessions subjects respond-

Table 1

Order of deprivation conditions and number of sessions in each condition (DPW = delayed postsession watering).

Rat	Phase	Deprivation (hr)	Sessions
R1	1	6	13
		2	18
		0	17
		23.5	13
		47	37
		4	15
	2	6	12
		4	12
		18	15
		12	13
	DPW	12	12
		6	12
R2	1	2	17
		23.5	27
		4	22
		47	17
		6	27
		0	25
	2	6	37
		18	17
		12	36
		12	12
	DPW	6	19
		12	12
R14	1	4	12
		6	24
		0	14
		23.5	15
		2	13
		47	21
	2	6	22
		18	14
		12	12
		12	19
	DPW	6	17
		12	12
R16	1	0	14
		2	25
		47	12
		4	16
		6	20
		23.5	14
	2	6	14
		18	20
		12	20
		12	30
	DPW	6	12
		12	12
R19	1	23.5	14
		4	27
		47	12
		2	22
		6	14
		0	12
	2	6	12
		18	14
		12	24
		12	13
	DPW	6	13
		12	13

ed on one of five VI schedules for 540 s, then received a 300-s blackout, and then responded on the next schedule for 540 s, and so on until all schedules were presented. The mean interval durations for the VI schedules were 5, 10, 30, 75, and 150 s in both phases of the experiment. For each schedule the programmed interreinforcement intervals were determined by Fleshler and Hoffman's (1962) method. The sequence of schedules within each session was random without replacement. This procedure was identical to that used by Heyman and Monaghan (1987).

Each VI schedule was associated with a unique set of stimuli. The stimulus conditions were the same as those used by Heyman and Monaghan (1987), except that a 1000-Hz tone was used instead of a clicker. For all five schedules, the left stimulus light was illuminated continuously. For the VI 150-s schedule, no additional discriminative stimulus was arranged. For the VI 75-s schedule, the right stimulus light flashed on for 0.2 s and off for 2.5 s (slow flash). For the VI 30-s schedule, the tone flashed on for 0.2 s and off for 2.5 s (slow flash). For the VI 10-s schedule, the right stimulus light and the tone simultaneously flashed on for 0.2 s and off for 1.5 s (medium flash). Finally, for the VI 5-s schedule, the right stimulus light and the tone simultaneously flashed on for 0.2 s and off for 0.25 s (fast flash). At the end of each VI component (i.e., during the blackouts), all stimulus lights and the houselight were extinguished, the tone was silent, and lever pressing had no effect.

*Reinforcement.* Reinforcement consisted of a 0.025-ml bead of water dropped into the recessed pan. During reinforcement the interval timer, the session timer, the stimulus lights, and the tone were inoperative. Only the houselight remained illuminated. The duration of this reinforcement period was held constant at 4.0 s throughout the experiment.

*Delayed postsession watering conditions.* Two follow-up conditions were arranged to assess the effects of immediate postsession watering on responding. The first condition was a 12-hr deprivation condition in which postsession water was delayed 5 hr. In this condition, 5 hr after each session rats were given 6 hr of free access to water, followed by 12 hr of water deprivation before the start of the next

session. The second condition was a 6-hr deprivation condition in which postsession water was delayed 11 hr. Eleven hours after each session, rats were given 6 hr of free access to water, followed by 6 hr of water deprivation before the start of the next session. All rats received the 12-hr deprivation condition first, followed by the 6-hr deprivation condition. The VI series, discriminative stimuli, method of schedule presentation, and reinforcement used in Phases 1 and 2 were also used in the delayed postsession watering conditions.

## RESULTS

Rats R2, R14, R16, and R19 did not respond in the 4-hr deprivation condition in Phase 2. The other deprivation conditions, including the delayed postsession watering conditions, remained in effect for a minimum of 12 sessions and continued until responding was stable. Stability was determined by time-series analysis of response rates in eight-session blocks using Young's (1941) C statistic, which detects trends in ordered sequences ( $\alpha = 0.10$ ; Tryon, 1982). In all cases the statistical judgment of stability was confirmed by visual inspection. For all rats, stability was achieved within 37 sessions in Phases 1 and 2, and within 30 sessions in the follow-up conditions. The number of sessions in each condition is listed in Table 1. All rats in all deprivation conditions that supported responding engaged in uninterrupted drinking for at least 5 min following each experimental session, indicating that satiation did not occur in any session.

Reinforcement and response rates were averaged over the last eight sessions at each VI value in each condition. The average rates in Phases 1 and 2, and their standard errors, are listed in Appendix B. Equation 1 was fitted to the average reinforcement and response rates by the method of least squares (McDowell, 1981). The resulting estimates of  $k$  and  $r_0$ , their standard errors, and the percentages of variance accounted for by the fits are listed in Table 2. Equation 1 provided an excellent description of these rats' response rates. For 45 of the 46 fits, the equation accounted for at least 90% of the response-rate variance. Residuals were examined by plotting the standardized residual against the predicted response rate, as recommended by Pedhazur



Table 2

Estimates of  $k$  and  $r_e$  and the percentage of variance accounted for (%VAF) by fits of Equation 1 to average response and reinforcement rates for individual rats in Phases 1 and 2. The standard errors of the estimates of  $k$  and  $r_e$  are given in parentheses.

Rat	Deprivation (hr)	$k$	$r_e$	%VAF	Average amount of water per session (ml)
<b>R1</b>					
Phase 1	47	57.3 (4.6)	53.3 (15.8)	95	2.83
	23.5	80.5 (7.3)	97 (28.1)	96	2.92
	6	83.4 (7.5)	213.2 (47.3)	99	2.72
	4	63.9 (10.8)	127.4 (63.7)	93	2.83
	2	93.8 (16.5)	521.1 (159)	99	2.61
	0	36.5 (7.6)	278.2 (104.4)	98	1.73
Phase 2	18	60.2 (15.8)	376.6 (189.5)	97	2.60
	12	79.8 (10)	588.7 (120.5)	100	2.56
	6	52.5 (8)	475.7 (113.9)	100	2.26
	4	30.5 (18.1)	391.4 (329.8)	97	1.28
<b>R2</b>					
Phase 1	47	58.4 (3.7)	102.8 (16.8)	99	2.94
	23.5	97.2 (5.5)	202.1 (29.4)	99	2.92
	6	100.5 (11.6)	427.7 (96.3)	99	2.85
	4	86 (9.8)	309.5 (76.6)	99	2.87
	2	91 (13.9)	433.3 (124)	99	2.71
	0	78.6 (30)	830.3 (436.7)	99	2.15
Phase 2	18	84.3 (12.3)	706.9 (161)	100	2.55
	12	47.2 (6.1)	394.7 (92.4)	99	2.38
	6	53.2 (10.6)	724.2 (196.7)	100	1.71
<b>R14</b>					
Phase 1	47	19.6 (1.5)	14.4 (6.3)	78	2.67
	23.5	17.1 (1)	24 (6.5)	92	2.61
	6	19.1 (2.4)	96.2 (32.1)	93	2.32
	4	21.8 (2.7)	132.4 (41.1)	96	2.38
	2	16 (1.7)	162.5 (40.6)	98	2.12
	0	4.2 (1)	36.7 (15.6)	90	0.91
Phase 2	18	15.9 (1.2)	141 (26.5)	99	2.23
	12	13.8 (2.2)	104.2 (44.6)	94	2.12
	6	5.5 (1)	94.5 (26.9)	98	0.91
<b>R16</b>					
Phase 1	47	102 (16)	339.2 (112.3)	98	2.91
	23.5	115.8 (27)	969.2 (323.9)	99	2.62
	6	30.7 (2.8)	267.1 (45.8)	100	1.99
	4	24.8 (4.6)	308.9 (93.3)	99	1.80
	2	25.7 (12.1)	347.2 (243.2)	94	1.51
	0	40.4 (17.9)	511.8 (307.9)	99	1.33
Phase 2	18	72.6 (6.9)	1,105.7 (130.4)	100	1.29
	12	12 (1.5)	188.7 (38.1)	99	1.18
	6	3.7 (2)	129.4 (90.9)	98	0.30
<b>R19</b>					
Phase 1	47	86.2 (7.4)	230.2 (48.3)	99	2.95
	23.5	85.8 (12.2)	230.2 (79.2)	97	2.94
	6	97.4 (26.6)	709.4 (312.7)	99	2.72
	4	71.7 (12.2)	361.4 (121.6)	99	2.68
	2	77.8 (16.1)	462.6 (177.9)	98	2.72
	0	53.4 (11.8)	433.6 (157)	99	2.01
Phase 2	18	64.4 (12.9)	577.4 (186.5)	99	2.36
	12	24.3 (5.1)	298.5 (95.4)	99	1.01
	6	22.1 (12)	548.4 (381.3)	99	0.88

(1982). Regression analyses of these residuals revealed no significant linear, quadratic, cubic, quartic, quintic, or hexic polynomial trends for any rat, or for the residuals pooled across rats. In addition, the residuals appeared to be homoscedastic, and were uncorrelated with obtained reinforcement rates (average Pearson  $r = -0.09$ ). There were no outlier residuals (defined as  $z > 2$ ) and, contrary to Baum's (1993) finding, there was no evidence of an upturn in response rates at high reinforcement rates. These results indicate that the assumptions required to fit Equation 1 by the method of least squares were met, and that the rats' response rates showed no systematic deviation or any other unusual departure from Equation 1.

The 6-, 23.5-, and 47-hr deprivation conditions in Phase 1 constituted a direct replication of Heyman and Monaghan's (1987) Experiment 3. Most of the procedural details in Phase 1 were identical, or nearly identical, to those in Heyman and Monaghan's experiment, including the method of water deprivation, the VI values, the method of schedule presentation, the discriminative stimuli associated with the schedules, the size of the reinforcer, and the effortful force requirement on the lever. There were four differences between the experiments: (a) Each used a different strain of rat, (b) Heyman and Monaghan used a clicker instead of a tone as part of the discriminative stimuli, (c) Heyman and Monaghan used a slightly higher force requirement on the lever, and (d) Heyman and Monaghan's most severe deprivation condition lasted 47.5 instead of 47 hr.

Heyman and Monaghan (1987) reported the median  $k$  across rats in each deprivation condition. Their  $k$ s are plotted in Figure 1 as open circles connected by dashed lines. The median  $k$  across rats from the comparable Phase-1 deprivation conditions are plotted as filled circles connected by solid lines. The results from the comparable conditions of the present experiment closely approximated those from Heyman and Monaghan's experiment. In both cases the  $k$ s were roughly constant from 6 to 23.5 hr of water deprivation. The lower  $k$ s for the 47- and 47.5-hr deprivation conditions may have been due to the compromised health of the rats in these conditions. As noted earlier, all rats in the present experiment lost weight rapidly in the 47-

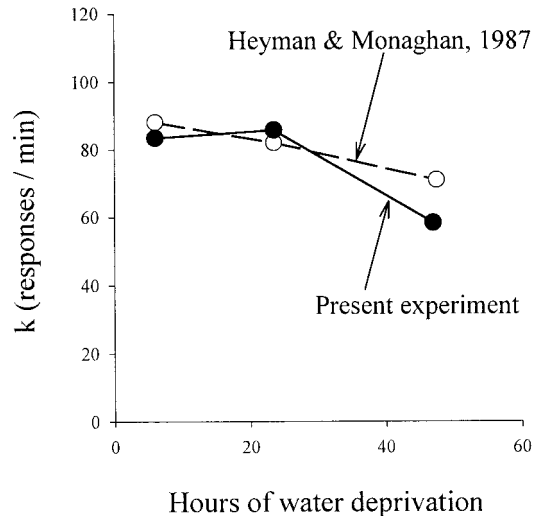


Fig. 1. Median  $k$ s from Heyman and Monaghan's (1987) experiment (open circles, dashed line) and median  $k$ s from comparable conditions of the present experiment (filled circles, solid line) plotted as a function of hours of water deprivation.

hr deprivation conditions, and 3 of the 5 rats required periodic suspension of the water-deprivation regime. Weight loss ranged from about 9% to about 17% of free-feeding body weight during each bout of weight loss in this condition. Although Heyman and Monaghan did not report weight loss in their rats, other experimenters have noted weight loss in rats under extreme water-deprivation conditions (e.g., Christensen-Szalanski, Goldberg, Anderson, & Mitchell, 1980; Fallon, 1965; Finan, 1940).

The  $k$ s for individual rats in Phase 1 of the present experiment are plotted as filled circles in Figure 2. The mean  $k$  across rats is plotted in the bottom right panel. The  $k$ s for the 47-hr deprivation condition are omitted and will not be considered further because of the possible confounding effect of weight loss and compromised health in this deprivation condition. The two rightmost points (filled circles) in each panel show the  $k$ s for the 6- and 23.5-hr conditions. Consistent with the median  $k$ s in Figure 1, these individual-subject  $k$ s are roughly the same in the two conditions for 4 of the 5 rats. However, the three leftmost points (filled circles) in each panel show that  $k$  decreased with decreasing deprivation level for those same 4 rats. Considering all Phase 1 conditions plotted in Figure 2,  $k$

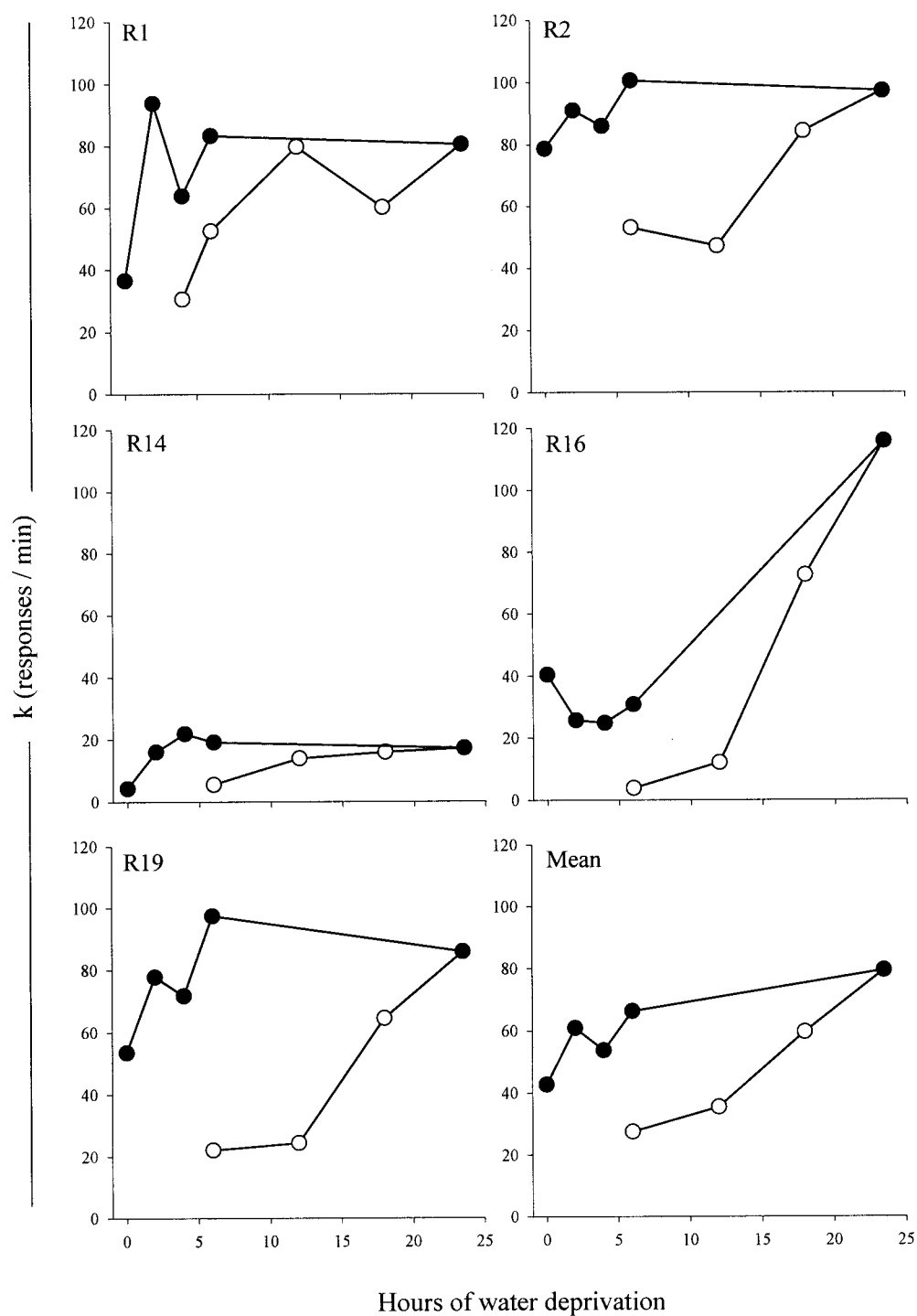


Fig. 2. Herrnstein's  $k$  from Phase 1 (filled circles) and Phase 2 (open circles) plotted as a function of hours of water deprivation for each rat. Mean  $k$ s across rats are plotted in the bottom right panel.



increased from the least to the most severe deprivation condition for all rats. The increase in  $k$  ranged from 24% for R2 to 307% for R14. The median percentage increase in  $k$  was 120% (R1), which is consistent with the approximately 100% increase in the mean  $k$  across rats (bottom right panel).

The  $k$ s for individual rats in Phase 2 are plotted as open circles in Figure 2, along with the mean  $k$  across rats for this phase. Recall that the Phase 2 deprivation conditions were less severe than the Phase 1 conditions having the same nominal values. The vertical separation of the Phase 1 and Phase 2  $k$ s in each panel shows that the more severe Phase 1 deprivation conditions consistently produced larger  $k$ s. The 6-hr deprivation conditions can be compared directly across the two phases. As shown in Figure 2, the 6-hr deprivation condition in Phase 1 yielded a larger  $k$  than the 6-hr deprivation condition in Phase 2 for every rat. The mean  $k$  for this condition in Phase 1 was 142% larger than the mean  $k$  in Phase 2. A marked change in  $k$  was also evident within the Phase 2 conditions. All rats showed an increase in  $k$  from the least to the most severe deprivation level in this phase.

The estimated values of  $r_e$  also varied with deprivation level. In Phase 1,  $r_e$  tended to decrease with deprivation level for 4 of the 5 rats. The exception was R16, whose  $r_e$  increased with deprivation level. The function form relating  $r_e$  to hours of deprivation in Phase 2, including the 24-hr condition from Phase 1, was more variable. For R1 and R2 it was decreasing, for R14 and R19 it was roughly flat, and for R16 it was increasing.

Data from the delayed postsession watering conditions were averaged over the last eight sessions at each VI value. The average reinforcement and response rates and their standard errors are listed in Appendix C. Equation 1 was fitted to the average reinforcement and response rates by the method of least squares (McDowell, 1981). The resulting estimates of  $k$  and  $r_e$ , their standard errors, and the percentages of variance accounted for by the fits are listed in Table 3. As was the case for Phases 1 and 2, Equation 1 provided an excellent description of the rats' response rates, accounting for nearly all the response-rate variance in the majority of cases.

Differences between response rates in the delayed and immediate (Phase 2) postsession

Table 3

Estimates of  $k$  and  $r_e$  and the percentage of variance accounted for (%VAF) by fits of Equation 1 to average response and reinforcement rates for individual rats in the delayed postsession watering conditions. The standard errors of the estimates are given in parentheses.

Rat	Deprivation (hr)	$k$	$r_e$	%VAF
R1	12	83 (14.7)	564.1 (72.1)	99
	6	12.8 (1.7)	159.8 (41.6)	99
R2	12	54.6 (11.5)	464.7 (170.4)	99
	6	57.3 (37.1)	1159.1 (855.2)	99
R14	12	12.4 (1.2)	128.9 (29)	99
	6	2.1 (0.4)	29.4 (14.7)	88
R16	12	14.7 (9)	155.6 (184.9)	91
	6	4.1 (4)	121.3 (147.5)	92
R19	12	53.1 (11.7)	335.7 (147.6)	98
	6	5 (1.3)	158.3 (54.8)	99

watering conditions at 12 and 6 hr of water deprivation varied around zero, and in most cases were small. The median difference across VIs and rats between the two postsession watering conditions at 12 hr of deprivation was 0 responses per minute. The median difference across VIs and rats at 6 hr of water deprivation was 1 response per minute, with the higher median response rate occurring in the immediate postsession watering condition (Phase 2). Independent  $t$  tests (assuming unequal variances and  $\alpha = 0.05$ ) on each rat's response rates at 12 hr and at 6 hr of deprivation showed no significant differences between responding in the delayed and immediate postsession watering conditions for any rat. Clearly, there was no indication that immediate postsession watering suppressed response rate.

## DISCUSSION

The results of Heyman and Monaghan's (1987) Experiment 3 were replicated in Phase 1 of the present experiment. The  $y$  asymptote,  $k$ , of Herrnstein's hyperbola (Equation 1), remained roughly constant, at least for the 6- and 23.5-hr deprivation conditions. However, when the range of water-deprivation conditions, and hence reinforcer magnitudes, was extended to include less severe levels and hence smaller reinforcer magnitudes,  $k$  was found to decline markedly. As shown in Figure 2, the change in  $k$  was con-

sistent across rats and methods of water deprivation. Furthermore, the change in  $k$  could not have been an artifact of the interaction between immediate postsession watering and deprivation level, which might have produced relatively more suppression of response rates at low than at high levels of water deprivation. Not only was no interaction between immediate postsession watering and deprivation level found, but no effect at all of immediate postsession watering was found (cf. Staddon & Ettinger, 1989; Timberlake, 1984; Timberlake, Gawley, & Lucas, 1987).

The change in  $k$  shown in Figure 2 violates the strongest form of Herrnstein's matching theory, which requires  $k$  to remain constant across changes in reinforcer magnitude (Herrnstein, 1974; McDowell, 1986). As noted earlier, if  $k$  is not constant, then Equation 1 cannot be obtained algebraically from the original matching equation (Herrnstein, 1970). These results do not, however, invalidate the form of Equation 1. On the contrary, there is overwhelming evidence that the form of the relationship between response and reinforcement rates is very nearly, if not exactly, hyperbolic (e.g., deVilliers & Herrnstein, 1976; McDowell, 1988; present experiment, Tables 1 and 2). These results also do not falsify a purely algebraic form of matching theory (McDowell, 1986), which permits  $k$  to vary, nor do they bear on matching theory's account of concurrent schedules.

It may be worthwhile to reconsider the data from McDowell and Wood (1984, 1985), who directly and systematically investigated Herrnstein's constant  $k$  assumption. According to McDowell and Wood, their data showed consistent violations of this assumption. But Heyman and Monaghan (1987) argued that a straight line often described McDowell and Wood's data better than a hyperbola: "In sum, the relation between response rate and reinforcement rate often did not conform to the predictions of Equation 1 [a hyperbola], and under these conditions, conclusions based on Equation 1 may be of questionable value" (Heyman & Monaghan, 1987, p. 392). It is important to recognize that any set of data with a true monotonic increasing trend will be reasonably well described by a variety of monotonically increasing function forms, including a straight line. The appropriate question is not whether one or another func-

tion form can describe a set of data, but whether a particular form describes the data best. In the case of McDowell and Wood's data, for example, fits of Equation 1 to their 68 individual-subject data sets can be compared to fits of a straight line. For a more stringent test of whether McDowell and Wood's data are best described by a hyperbola, other monotonically increasing function forms that are much closer to hyperbolic in form can be fitted to their data (cf. deVilliers & Herrnstein, 1976). Two good choices are the asymptotic exponential,

$$R = m(1 - e^{-r/b}), \quad (3)$$

and the asymptotic power function,

$$R = m[1 - (r + 1)^{-b}], \quad (4)$$

where  $R$  and  $r$  represent response and reinforcement rates, and  $m$  and  $b$  are parameters of the equations. Like a hyperbola, both of these equations pass through the origin, are concave downward, and have  $y$  asymptotes in the first quadrant. In addition, their differential properties are very similar to those of a hyperbola (e.g., given appropriate parameter values, the functions increase rapidly from the origin and have continuously decreasing first derivatives in the first quadrant). None of these hyperbola-like features is characteristic of a straight line.

Equations 3 and 4 were fitted to the 68 individual-subject data sets from McDowell and Wood's (1984, 1985) experiments using a least squares criterion. The percentages of variance accounted for by a line and by Equations 3 and 4 were then compared to the percentages of variance accounted for by a hyperbola using Wilcoxon's matched-pairs signed-ranks test. The results of this comparison are shown in Table 4. As expected, all function forms provided reasonably good descriptions of the data, and Equations 3 and 4 provided better descriptions than a straight line. But as shown in the first column of Table 4, a hyperbola on average accounted for an additional 8% of the data variance when compared to a line, an additional 5% of the variance when compared to Equation 3, and an additional 3% of the variance when compared to Equation 4. Moreover, the superiority of the hyperbola was statistically significant in each case. Hence, a hyperbola not only described McDowell and Wood's data

Table 4

Results of Wilcoxon's matched pairs signed ranks test comparing fits of a hyperbola (Equation 1) to fits of a line, an asymptotic exponential (Equation 3), and an asymptotic power function (Equation 4) to the 68 data sets from McDowell and Wood (1984, 1985). The first column shows the mean difference between the percentage of variance accounted for (%VAF) by a hyperbola and the %VAF by the comparison form. Positive values indicate greater %VAF by the hyperbola. Values of  $T$ ,  $z$ , and  $p$  (two tailed) are given for Wilcoxon's matched pairs signed ranks test. The significant  $p$  values show that the hyperbola accounted for a greater percentage of the variance of the 68 data sets than did the other forms.

Comparison	Mean difference in %VAF	$T$	$z$	$p$
Hyperbola vs. line	8.12	825	-2.13	< .05
Hyperbola vs. exponential	5.24	461	-4.35	≪ .01
Hyperbola vs. power function	3.16	725	-2.74	< .01

better than a line, but it also described their data better than two functions with hyperbola-like forms.

This analysis suggests that Heyman and Monaghan's (1987) concern about McDowell and Wood's (1984, 1985) data was unwarranted. Their data are in fact well described by Equation 1, and hence it seems reasonable to accept their estimates of  $k$  as valid. This means that the results of McDowell and Wood's experiments, Heyman and Monaghan's experiment, and the present experiment are all consistent: Herrnstein's  $k$  varies over an appropriately selected range of reinforcer magnitudes, in direct violation of matching theory. In other words, these experiments contradict Herrnstein's constant  $k$  assumption, and hence cast doubt on the original theoretical foundation of Equation 1.

Given the results of these four experiments (Heyman & Monaghan, 1987; McDowell & Wood, 1984, 1985; and the present experiment), it is not surprising that findings from other research bearing on Herrnstein's constant  $k$  assumption are mixed. But to falsify the strongest form of matching theory, it is only necessary to show that  $k$  varies reliably over *some* range of reinforcer magnitude, and the evidence for this is clear. Failures to find a variable  $k$  may be due to inadequate sampling of the magnitude range, as in Heyman and Monaghan (1987), or to using a response with too small a cost (McDowell, 1980; McDowell & Wood, 1985), or to varying a property of the reinforcer that does not affect its value, or to some combination of these reasons.

### *Falsifiability*

Even in the strongest form of matching theory,  $k$  is permitted to vary when the instrumental response changes (Herrnstein, 1974). For example, the value of  $k$  can be different for a pigeon pecking a key on a series of VI schedules than for a pigeon pressing a treadle on the same series of VI schedules. In one case all behavior is measured in terms of a key peck, and in the other it is measured in terms of a treadle press.

Finding different  $k$ s in an experiment might lead one to check the responses to make sure they were the same in the various conditions. It seems that if they were found to be the same in every obvious way, then one could conclude that the theory is false. But it is possible to argue instead that the responses might have differed in some subtle way, thus generating a theoretically acceptable variation in  $k$ . For example, perhaps the initial blow on the lever was more forceful in one condition than in another. One might then set up recording equipment to detect the initial force on the lever. It seems that if no difference in initial force were detected, then one could assert that the responses did not differ and conclude that the theory must be false. But it is possible to argue instead that an even more subtle difference might have caused the  $k$ s to vary in a manner consistent with the theory. Perhaps, for example, the behavior was psychologically more onerous in one condition than in another. This argument can continue indefinitely. In spite of any data that might be collected, ever more

subtle differences in behavior could be invoked to account for the different  $k$ s. From this point of view, no type or amount of evidence can falsify matching theory.

The Charybdis of unfalsifiability arises because it is not possible to prove the null condition, namely, that the responses are the same; it is only possible to detect a difference. Unfalsifiability is not usually considered a virtue in a scientific theory (e.g., Herrnstein, 1969). To avoid succumbing to it in this instance, reasonable steps must be taken to ensure that the responses are the same. We agree with Herrnstein (1974), that when instrumental behavior reasonably can be said to remain unchanged, a variable  $k$  falsifies matching theory. In all deprivation conditions of the present experiment, the same lever with the same force requirement was used, and no differences were noted in the form of the rats' lever pressing (although no detailed data, such as force measurements, were collected). We believe that our finding of a variable  $k$  under these circumstances indicates that the strongest form of matching theory is false.

#### *Linear System Theory*

As mentioned earlier, linear system theory (McDowell, 1987; McDowell et al., 1983, 1993; McDowell & Kessel, 1979) has produced an equation (Equation 2) relating response and reinforcement rates. McDowell (1980) showed that the form of this equation is indistinguishable from the hyperbolic form of Equation 1. This means that the well-documented hyperbolic relationship between response and reinforcement rates is described equally well by Equations 1 and 2. Of course, the theoretical basis of Equation 2 is very different from that of Herrnstein's hyperbola. It is based on two straightforward observations, namely that reinforcement and response events occur in time, and that present behavior depends on past reinforcement events (McDowell et al., 1993). An important mathematical consequence of the linear system theory is that it requires Herrnstein's  $k$  to vary with reinforcer magnitude (McDowell, 1980). McDowell and Wood (1984, 1985) found that the form of the variation in  $k$  they observed was consistent with the form required by linear system theory. Like McDowell and Wood's data, the present data also

bear on the linear system theory's prediction regarding Herrnstein's  $k$ . According to the theory,  $k$  is required to vary approximately with reinforcer value,  $P_R$ , according to

$$k \cong \frac{-mP_R}{P_R - b}, \quad (5)$$

where  $m$  and  $b$  are parameters of the equation (McDowell, 1980). The derivation of Equation 5 is given in Appendix A. The equation has a negative  $y$  asymptote, a positive  $x$  asymptote, and is concave upward in the first quadrant. According to this unusual and counterintuitive function form,  $k$  should increase with  $P_R$  in the first quadrant, and should do so more rapidly as  $P_R$  gets large, until finally  $P_R$  exceeds the positive  $x$  asymptote of Equation 5. At this point responding suddenly goes to zero (in the region of negative  $k$ s).

The principal problem in applying Equation 5 is deciding what quantities to use for reinforcer value,  $P_R$ . For the present data, the nominal deprivation values are clearly inappropriate because different deprivation methods were used in the two phases of the experiment. It is interesting to note that this problem, namely determining the relationship between the reinforcing value of water and the nominal deprivation level, is a problem in psychophysics. One possible solution to this problem is to use the average amount of water consumed per session at a given deprivation level as an estimate of the reinforcing value of water in that condition. In other words, it seems reasonable to suppose that given equal opportunity and constraint (i.e., identical session lengths and schedule values in all deprivation conditions), the amount of water consumed is a measure of its value. This measure of reinforcer value has good face validity, but it also might be worthwhile to ask whether it is consistent with Stevens' psychophysical power law (Herrnstein & Brown, 1975), which states that the judged or effective magnitude of a stimulus is a power function of its physical magnitude. To answer this question, a power function was fitted to data relating the amount of water consumed to the nominal deprivation level for each rat in each phase of the experiment. The average amount of water consumed at each deprivation level (average number of reinforcers

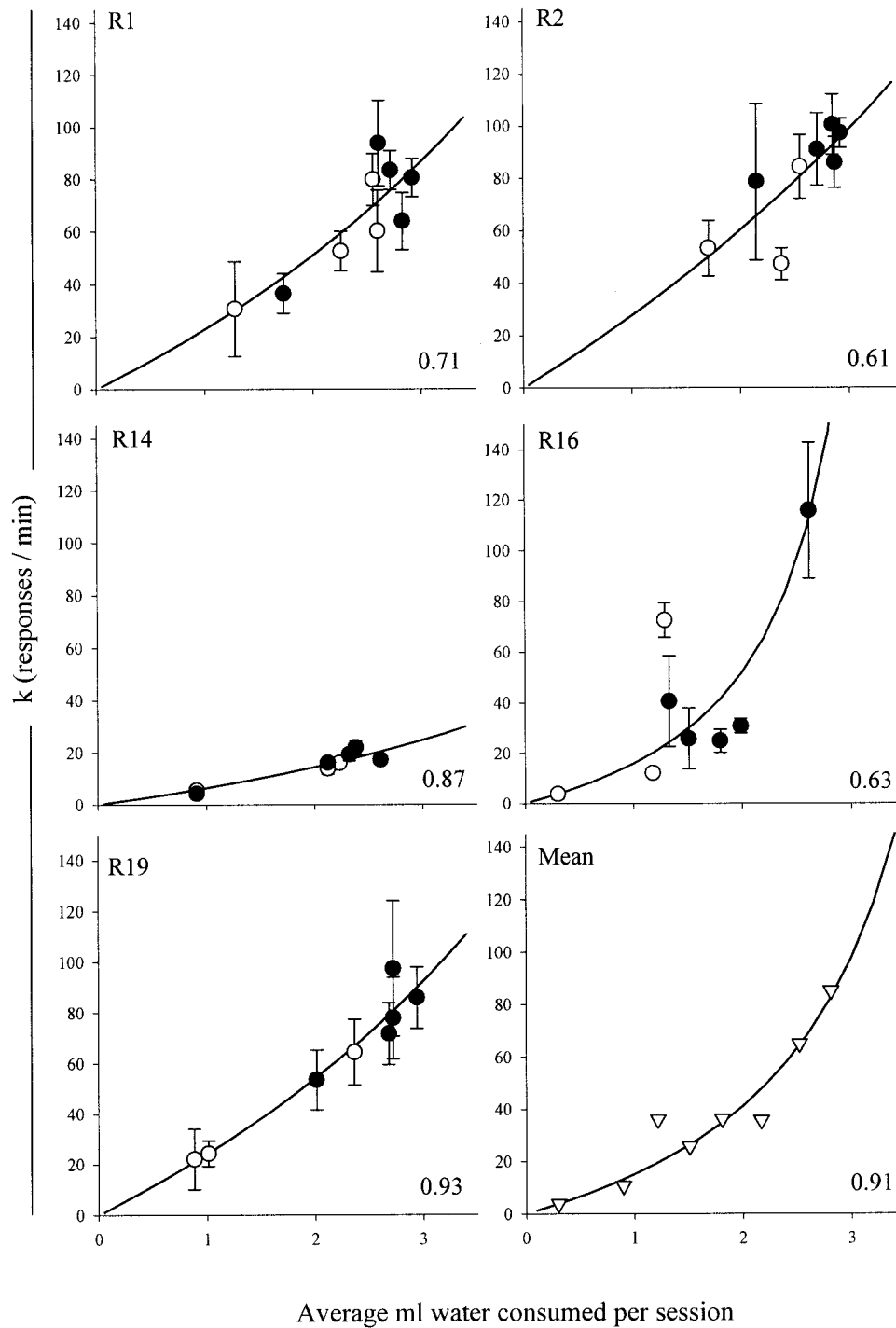


Fig. 3. Herrnstein's  $k$  for each rat from Phase 1 (filled circles) and Phase 2 (open circles) plotted as a function of average amount of water consumed per session in each deprivation condition. Error bars represent  $\pm 1$  standard error. Mean  $k$ s across rats are plotted in the bottom right panel. The method of obtaining the means is described in the text. The smooth curve is the least squares fit of Equation 5. The proportion of variance accounted for by the equation is given in the lower right corner of each panel.



times 0.025 ml per reinforcer) is listed in the last column of Table 2. A power function provided a good description of the relation between water consumed and nominal deprivation level in each phase of the experiment. For the rats in Phase 1, in numerical order, a power function accounted for 97%, 97%, 98%, 71%, and 98% of the variance of the average amount of water consumed per session. When water consumption was averaged across rats in Phase 1, a power function accounted for 100% of the variance in water consumed. The fits in Phase 2 were also good, but with only three data points for most rats, they are not good tests of the power function form. These results support the view that the average amount of water consumed per session is a reasonable measure of reinforcer value.

To examine the validity of Equation 5, the  $k$ s from both phases of the experiment are plotted in Figure 3 against the average amount of water consumed. The average across rats is shown in the bottom right panel and was obtained by dividing the  $x$  axis from 0 to 3 ml into nine equal segments, and then averaging the  $x$  values within each segment and the  $y$  values within each segment. Eight data points appear in this panel because the second segment contained no data. The plots in Figure 3 show that the average amount of water consumed in Phase 2 tended to be less than the average amount consumed in Phase 1. This is consistent with the view that reinforcer value tended to be lower in the less severe Phase 2 conditions. Notice also that the increases in  $k$  as reinforcer value increased were quite large for every rat. The increase in  $k$  from the lowest to the highest reinforcer value ranged from 83% for R2 to over 30-fold for R16. For the average data,  $k$  increased more than 20-fold from the lowest to the highest reinforcer value. These plots are consistent with those in Figure 2, but they present a different and more striking view of the change in Herrnstein's  $k$  because of the common metric for reinforcer value. The smooth curve drawn through the points in each panel is the least squares fit of Equation 5. The number in the lower right corner of each panel gives the proportion of variance accounted for by the fit. The approximate function form required by linear system theory described these somewhat noisy data rea-

sonably well. Hence, consistent with the findings of McDowell and Wood (1984, 1985), the change in  $k$  in the present experiment agreed not only qualitatively but also quantitatively with the prediction of linear system theory.

These results suggest that Equation 2 may be a reasonable candidate for replacing Equation 1. Its form is very nearly hyperbolic (McDowell, 1980), as any reasonable candidate equation must be, and it both qualitatively and quantitatively predicts the violation of matching theory found in the present experiment and in McDowell and Wood's (1984, 1985) experiments. Equation 2 also has the advantage of expressing response rate as a joint function of reinforcement rate ( $R_{in}$ ), reinforcer value ( $P_R$ ), and response aversiveness ( $P_B$ ), all of which are known to affect response rate (Baum & Rachlin, 1969; Chung, 1965; Ferster & Skinner, 1957). McDowell (1987) showed that the relationship between response rate and reinforcer value specified by Equation 2 describes existing data well, and McDowell and Kessel (1979) showed that at least one data set was consistent with the equation's specification of how response rate varies jointly with reinforcement rate and reinforcer value. Further advances in the application of Equation 2 may require a more careful consideration of the psychophysics of reinforcer value and the psychophysics of response aversiveness, so that unambiguous values can be assigned to  $P_R$  and  $P_B$ .

## REFERENCES

- Aseltine, J. A. (1958). *Transform method in linear system analysis*. New York: McGraw-Hill.
- Baum, W. M. (1993). Performances on ratio and interval schedules of reinforcement: Data and theory. *Journal of the Experimental Analysis of Behavior*, 59, 245-264.
- Baum, W. M., & Rachlin, H. C. (1969). Choice as time allocation. *Journal of the Experimental Analysis of Behavior*, 12, 861-874.
- Bradshaw, C. M., Ruddle, H. V., & Szabadi, E. (1981). Relationship between response rate and reinforcement frequency in variable interval schedules: II. Effect of the volume of sucrose reinforcement. *Journal of the Experimental Analysis of Behavior*, 35, 263-270.
- Bradshaw, C. M., Szabadi, E., & Bevan, P. (1978). Relationship between response rate and reinforcement frequency in variable-interval schedules: The effect of concentration of sucrose reinforcement. *Journal of the Experimental Analysis of Behavior*, 29, 447-452.



- Brown, B. M. (1961). *The mathematical theory of linear systems*. New York: Wiley.
- Campbell, B. A., & Kraeling, D. (1953). Response strength as a function of drive level and amount of drive reduction. *Journal of Experimental Psychology*, 45, 97–101.
- Christensen-Szalanski, J. J., Goldberg, A. D., Anderson, M. E., & Mitchell, T. R. (1980). Deprivation, delay of reinforcement, and the selection of behavioural strategies. *Animal Behaviour*, 28, 341–346.
- Chung, S. (1965). Effects of effort on response rate. *Journal of the Experimental Analysis of Behavior*, 8, 1–7.
- deVilliers, P. (1977). Choice in concurrent schedules and a quantitative formulation of the law of effect. In W. K. Honig & J. E. R. Staddon (Eds.), *Handbook of operant behavior* (pp. 233–287). Englewood Cliffs, NJ: Prentice Hall.
- deVilliers, P. A., & Herrnstein, R. J. (1976). Toward a law of response strength. *Psychological Bulletin*, 83, 1131–1153.
- Fallon, D. (1965). Effects of cyclic deprivation upon consummatory behavior: The role of deprivation history. *Journal of Comparative and Physiological Psychology*, 60, 283–287.
- Ferster, C. B., & Skinner, B. F. (1957). *Schedules of reinforcement*. Englewood Cliffs, NJ: Prentice Hall.
- Finan, J. L. (1940). Quantitative studies in motivation: I. Strength of conditioning in rats under varying degrees of hunger. *Journal of Comparative Psychology*, 29, 119–134.
- Fleshler, M., & Hoffman, H. S. (1962). A progression for generating variable interval schedules. *Journal of the Experimental Analysis of Behavior*, 5, 529–530.
- Guttman, N. (1954). Equal-reinforcement values for sucrose and glucose solutions compared with equal-sweetness values. *Journal of Comparative and Physiological Psychology*, 47, 358–361.
- Herrnstein, R. J. (1969). Method and theory in the study of avoidance. *Psychological Review*, 76, 49–69.
- Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior*, 13, 243–266.
- Herrnstein, R. J. (1974). Formal properties of the matching law. *Journal of the Experimental Analysis of Behavior*, 21, 159–164.
- Herrnstein, R. J., & Brown, R. W. (1975). *Psychology*. Boston: Little, Brown.
- Heyman, G. M., & Monaghan, M. M. (1987). Effects of changes in response requirement and deprivation on the parameters of the matching law equation: New data and review. *Journal of Experimental Psychology: Animal Behavior Processes*, 13, 384–394.
- Heyman, G. M., & Monaghan, M. M. (1994). Reinforcer magnitude (sucrose concentration) and the matching law theory of response strength. *Journal of the Experimental Analysis of Behavior*, 61, 505–516.
- Keesey, R. E. (1962). The relation between pulse frequency, intensity, and duration and the rate of responding for intracranial stimulation. *Journal of Comparative and Physiological Psychology*, 55, 671–678.
- Keesey, R. E. (1964). Duration of stimulation and the reward properties of hypothalamic stimulation. *Journal of Comparative and Physiological Psychology*, 58, 201–207.
- Keesey, R. E., & Kling, J. W. (1961). Amount of reinforcement and free-operant responding. *Journal of the Experimental Analysis of Behavior*, 4, 125–132.
- Kraeling, D. (1961). Analysis of amount of reward as a variable in learning. *Journal of Comparative and Physiological Psychology*, 54, 560–565.
- Logan, F. A. (1960). *Incentive: How conditions of reinforcement affect the performance of rats*. New Haven, CT: Yale University Press.
- McDowell, J. J. (1980). An analytic comparison of Herrnstein's equations and a multivariate rate equation. *Journal of the Experimental Analysis of Behavior*, 33, 397–408.
- McDowell, J. J. (1981). Wilkinson's method of estimating the parameters of Herrnstein's hyperbola. *Journal of the Experimental Analysis of Behavior*, 35, 413–414.
- McDowell, J. J. (1986). On the falsifiability of matching theory. *Journal of the Experimental Analysis of Behavior*, 45, 63–74.
- McDowell, J. J. (1987). A mathematical theory of reinforcer value and its application to reinforcement delay in simple schedules. In M. L. Commons, J. E. Mazur, J. A. Nevin, & H. Rachlin (Eds.), *Quantitative analyses of behavior: Vol. 5. Reinforcement value: The effect of delay and intervening events* (pp. 77–105). Cambridge, MA: Ballinger.
- McDowell, J. J. (1988). Matching theory in natural human environments. *The Behavior Analyst*, 11, 95–109.
- McDowell, J. J., Bass, R., & Kessel, R. (1983). Variable-interval rate equations and reinforcement and response distributions. *Psychological Review*, 90, 364–375.
- McDowell, J. J., Bass, R., & Kessel, R. (1993). A new understanding of the foundation of linear system analysis and an extension to nonlinear cases. *Psychological Review*, 100(3), 407–419.
- McDowell, J. J., & Kessel, R. (1979). A multivariate rate equation for variable-interval performance. *Journal of the Experimental Analysis of Behavior*, 36, 9–19.
- McDowell, J. J., & Wood, H. M. (1984). Confirmation of linear system theory prediction: Changes in Herrnstein's  $k$  as a function of changes in reinforcer magnitude. *Journal of the Experimental Analysis of Behavior*, 41, 183–192.
- McDowell, J. J., & Wood, H. M. (1985). Confirmation of linear system theory prediction: Rate of change of Herrnstein's  $k$  as a function of response-force requirement. *Journal of the Experimental Analysis of Behavior*, 43, 61–73.
- Pedhazur, E. J. (1982). *Multiple regression in behavioral research: Explanation and prediction* (2nd ed.). New York: Holt, Rinehart, and Winston.
- Schrier, A. M. (1965). Response rates of monkeys (*Macaca mulatta*) under varying conditions of sucrose reinforcement. *Journal of Comparative and Physiological Psychology*, 59, 378–384.
- Snyderman, M. (1983). Body weight and response strength. *Behavior Analysis Letters*, 3, 255–265.
- Staddon, J. E. R., & Ettinger, R. H. (1989). *Learning: An introduction to the principles of adaptive behavior*. San Diego: Harcourt, Brace, Jovanovich.
- Timberlake, W. (1984). A temporal limit on the effect of future food on current performance in an analogue of foraging and welfare. *Journal of the Experimental Analysis of Behavior*, 41, 117–124.
- Timberlake, W., Gawley, D. J., & Lucas, G. A. (1987). Time horizons in rats foraging for food in temporally separated patches. *Journal of the Experimental Analysis of Behavior*, 13, 302–309.
- Tryon, W. W. (1982). A simplified time-series analysis for evaluating treatment interventions. *Journal of Applied Behavior Analysis*, 15, 423–429.

Warren-Boulton, F. R., Silberberg, A., Gray, M., & Ollom, R. (1985). Reanalysis of the equation for simple action. *Journal of the Experimental Analysis of Behavior*, 43, 265–277.

Williams, B. A. (1988). Reinforcement, choice, and response strength. In R. C. Atkinson, R. J. Herrnstein, G. Lindzey, & R. D. Luce (Eds.), *Stevens' handbook of experimental psychology* (2nd ed., Vol. 2, pp. 167–244). New York: Wiley.

Young, L. C. (1941). On randomness in ordered sequences. *Annals of Mathematical Statistics*, 12, 293–300.

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## APPENDIX A

McDowell (1980) showed that an expression for Herrnstein's  $k$  can be obtained from the series expansion approximation of the rate equation (Equation 2). The expression for  $k$ , with the power parameter substitutions explained in McDowell (1987), is

$$k \cong \left[ (e^{-w^*} - 1) - \frac{P_B e^{-w^*} (1 - e^w)}{\gamma P_R} \right]^{-1}.$$

Finding a common denominator and taking the reciprocal of the right side produces

$$k \cong \frac{\gamma P_R}{\gamma P_R (e^{-w^*} - 1) - P_B e^{-w^*} (1 - e^w)}.$$

Dividing top and bottom by  $\gamma(e^{-w^*} - 1)$ ,

$$k \cong \frac{\frac{1}{e^{-w^*} - 1} P_R}{P_R - \frac{P_B e^{-w^*} (1 - e^w)}{\gamma(e^{-w^*} - 1)}},$$

and letting

$$-m = \frac{1}{e^{-w^*} - 1} = \frac{e^{w^*}}{1 - e^{w^*}} \quad \text{and}$$

$$b = \frac{P_B e^{-w^*} (1 - e^w)}{\gamma(e^{-w^*} - 1)} = \frac{P_B (e^w - 1)}{\gamma(e^{w^*} - 1)},$$

we have

$$k \cong \frac{-m P_R}{P_R - b},$$

which is an expression for Herrnstein's  $k$  as a function of  $P_R$ . This equation relates Herrnstein's  $k$  to the series expansion approximation of the rate equation (Equation 2).

## APPENDIX B

Mean reinforcement and response rates in Phases 1 and 2. The number in parentheses is the standard error of the mean.

Deprivation (hr)			VI (seconds)				
			5	10	30	75	150
R1							
Phase 1	47	rft/hr	582 (7)	318 (6)	120 (7)	45 (5)	18 (4)
		rsp/min	48 (3)	55 (2)	39 (2)	26 (3)	14 (1)
	23.5	rft/hr	636 (12)	329 (5)	108 (4)	51 (6)	20 (3)
		rsp/min	65 (3)	69 (1)	40 (3)	27 (3)	15 (1)
	6	rft/hr	578 (25)	305 (17)	104 (4)	41 (4)	13 (4)
		rsp/min	59 (5)	53 (7)	26 (4)	13 (3)	4 (1)
	4	rft/hr	586 (7)	326 (6)	115 (8)	41 (3)	20 (4)
		rsp/min	47 (1)	54 (2)	29 (2)	12 (0.3)	9 (2)
	2	rft/hr	548 (16)	281 (10)	92 (5)	39 (3)	21 (2)
		rsp/min	49 (3)	30 (2)	15 (2)	9 (2)	4 (1)
	0	rft/hr	309 (50)	222 (21)	74 (8)	37 (4)	19 (2)
		rsp/min	18 (5)	18 (3)	7 (2)	4 (1)	3 (0.5)
Phase 2	18	rft/hr	512 (15)	293 (13)	98 (5)	35 (5)	26 (3)
		rsp/min	33 (4)	30 (5)	10 (1)	5 (1)	2 (0.1)
	12	rft/hr	522 (14)	263 (10)	93 (8)	39 (5)	28 (3)
		rsp/min	38 (1)	24 (3)	10 (1)	6 (1)	4 (1)

## APPENDIX B

(Continued)

Rat	Deprivation (hr)		VI (seconds)					
			5	10	30	75	150	
R2	6	rft/hr	443 (51)	247 (20)	84 (13)	31 (5)	14 (3)	
		rsp/min	25 (4)	18 (2)	7 (1)	4 (1)	2 (1)	
	4	rft/hr	214 (59)	133 (38)	45 (12)	21 (7)	13 (5)	
		rsp/min	10 (4)	9 (4)	2 (1)	1 (0.3)	1 (1)	
	Phase 1	47	rft/hr	635 (4)	323 (6)	113 (6)	52 (5)	26 (4)
		rsp/min	58 (2)	54 (2)	33 (2)	23 (1)	16 (1)	
	23.5	rft/hr	650 (7)	331 (7)	101 (5)	45 (3)	28 (5)	
		rsp/min	75 (2)	60 (2)	30 (2)	20 (2)	14 (2)	
	R14	6	rft/hr	624 (7)	330 (6)	101 (4)	36 (4)	25 (4)
			rsp/min	59 (1)	45 (2)	16 (2)	10 (1)	6 (1)
4		rft/hr	605 (10)	323 (8)	115 (3)	40 (5)	26 (3)	
		rsp/min	55 (2)	47 (3)	20 (3)	10 (1)	8 (1)	
2		rft/hr	556 (30)	310 (12)	97 (6)	39 (3)	26 (6)	
		rsp/min	50 (4)	41 (3)	14 (2)	8 (1)	4 (1)	
0		rft/hr	430 (37)	199 (12)	71 (8)	42 (3)	18 (4)	
		rsp/min	21 (4)	14 (2)	5 (1)	3 (0.4)	2 (0.2)	
Phase 2		18	rft/hr	541 (24)	259 (18)	88 (8)	39 (6)	27 (4)
		rsp/min	36 (3)	23 (3)	8 (1)	5 (1)	4 (0.5)	
12		rft/hr	450 (28)	263 (12)	77 (10)	37 (3)	27 (4)	
		rsp/min	25 (4)	20 (2)	7 (3)	4 (1)	3 (0.2)	
6		rft/hr	342 (54)	130 (29)	60 (8)	31 (4)	22 (3)	
		rsp/min	17 (4)	8 (3)	4 (1)	3 (1)	2 (0.2)	
Phase 1		47	rft/hr	517 (9)	293 (6)	112 (7)	44 (4)	22 (4)
		rsp/min	18 (1)	17 (1)	20 (1)	15 (2)	11 (1)	
23.5		rft/hr	462 (10)	286 (9)	112 (6)	51 (5)	27 (3)	
		rsp/min	15 (1)	16 (0.4)	15 (1)	11 (1)	8 (0.4)	
6		rft/hr	413 (31)	240 (13)	81 (12)	47 (2)	30 (4)	
		rsp/min	15 (1)	14 (2)	7 (1)	7 (1)	6 (1)	
4		rft/hr	452 (25)	242 (4)	84 (5)	42 (5)	28 (5)	
		rsp/min	17 (1)	13 (1)	7 (1)	6 (1)	5 (0.5)	
2		rft/hr	381 (57)	223 (10)	86 (10)	34 (6)	21 (5)	
		rsp/min	11 (2)	9 (1)	5 (1)	3 (1)	2 (0.3)	
0		rft/hr	115 (44)	73 (26)	51 (8)	25 (6)	15 (2)	
		rsp/min	3 (2)	2 (1)	3 (1)	2 (5)	1 (0.2)	
Phase 2	18	rft/hr	435 (26)	207 (24)	80 (8)	46 (6)	20 (3)	
	rsp/min	12 (1)	9 (1)	5 (1)	4 (1)	2 (0.4)		
R16	12	rft/hr	367 (35)	234 (13)	79 (6)	35 (4)	17 (3)	
		rsp/min	10 (1)	10 (1)	5 (1)	4 (1)	2 (1)	
	6	rft/hr	104 (40)	87 (18)	48 (9)	23 (5)	14 (3)	
		rsp/min	3 (1)	2 (1)	2 (0.3)	1 (0.3)	1 (0.3)	
	Phase 1	47	rft/hr	612 (6)	337 (8)	112 (5)	45 (6)	26 (5)
		rsp/min	65 (3)	53 (3)	20 (1)	15 (1)	10 (1)	
	23.5	rft/hr	545 (13)	275 (16)	93 (6)	43 (5)	26 (5)	
		rsp/min	41 (3)	26 (4)	8 (1)	6 (1)	4 (1)	
	6	rft/hr	356 (43)	203 (30)	75 (13)	32 (6)	21 (4)	
		rsp/min	18 (4)	13 (3)	6 (2)	3 (1)	3 (1)	
	4	rft/hr	251 (57)	217 (15)	72 (12)	32 (5)	28 (4)	
		rsp/min	11 (3)	11 (1)	4 (1)	2 (0.4)	2 (0.3)	
	2	rft/hr	244 (56)	110 (31)	50 (11)	34 (5)	27 (4)	
		rsp/min	11 (5)	6 (3)	2 (1)	3 (1)	3 (1)	
	0	rft/hr	230 (67)	133 (31)	41 (10)	23 (3)	16 (3)	
		rsp/min	12 (5)	9 (4)	2 (1)	2 (1)	1 (0.4)	

## APPENDIX B

*(Continued)*

Deprivation (hr)			VI (seconds)					
			5	10	30	75	150	
R19	Phase 2	18	rft/hr	322 (51)	97 (44)	56 (13)	15 (8)	15 (5)
			rsp/min	16 (3)	6 (3)	3 (1)	1 (0.4)	1 (0.3)
		12	rft/hr	187 (50)	106 (24)	45 (12)	23 (4)	15 (5)
			rsp/min	6 (2)	4 (1)	2 (1)	1 (0.3)	1 (0.3)
		6	rft/hr	47 (16)	21 (10)	7 (5)	8 (3)	2 (1)
			rsp/min	1 (0.3)	0.5 (0.2)	0.2 (0.2)	0.2 (0.1)	0.1 (0.1)
	Phase 1	47	rft/hr	638 (9)	341 (4)	119 (6)	44 (5)	19 (5)
				rsp/min	62 (2)	54 (2)	26 (2)	14 (1)
		23.5	rft/hr	624 (12)	332 (6)	118 (8)	48 (4)	25 (3)
				rsp/min	59 (3)	57 (2)	27 (3)	13 (1)
		6	rft/hr	594 (17)	284 (10)	104 (5)	44 (4)	18 (3)
				rsp/min	44 (3)	30 (6)	9 (1)	6 (1)
		4	rft/hr	541 (17)	302 (11)	93 (8)	48 (7)	25 (6)
				rsp/min	41 (4)	36 (5)	14 (4)	7 (2)
2		rft/hr	605 (11)	274 (14)	102 (5)	38 (6)	28 (4)	
			rsp/min	45 (3)	26 (5)	17 (3)	6 (1)	4 (1)
0		rft/hr	385 (67)	202 (42)	78 (8)	33 (5)	18 (5)	
			rsp/min	25 (7)	18 (7)	6 (2)	4 (2)	2 (1)
Phase 2	18	rft/hr	478 (51)	249 (33)	80 (14)	35 (7)	28 (5)	
			rsp/min	29 (5)	21 (5)	6 (2)	4 (1)	2 (0.2)
	12	rft/hr	182 (70)	113 (52)	40 (15)	12 (4)	4 (2)	
			rsp/min	7 (3)	9 (5)	2 (1)	1 (0.5)	0.3 (0.2)
	6	rft/hr	181 (32)	55 (23)	23 (13)	15 (6)	5 (2)	
			rsp/min	5 (1)	2 (1)	1 (0.4)	1 (0.5)	0.4 (0.1)

## APPENDIX C

Mean reinforcement and response rates in the delayed postsession watering conditions. The number in parentheses is the standard error of the mean.

Rat	Deprivation (hr)		VI (seconds)				
			5	10	30	75	150
R1	12	rft/hr	560 (12)	289 (14)	90 (5)	33 (8)	18 (4)
		rsp/min	41 (2)	29 (4)	9 (1)	6 (2)	3 (1)
	6	rft/hr	242 (37)	129 (26)	42 (10)	25 (7)	14 (4)
		rsp/min	8 (1)	6 (1)	2 (1)	2 (1)	1 (1)
R2	12	rft/hr	485 (39)	283 (15)	88 (10)	43 (5)	21 (5)
		rsp/min	27 (3)	22 (2)	7 (2)	5 (1)	4 (1)
	6	rft/hr	215 (47)	112 (30)	39 (10)	33 (3)	20 (2)
		rsp/min	9 (3)	5 (2)	1 (0.4)	1 (0.2)	1 (0.2)
R14	12	rft/hr	317 (20)	193 (31)	60 (11)	31 (6)	21 (5)
		rsp/min	8 (1)	8 (1)	4 (1)	2 (0.5)	2 (0.5)
	6	rft/hr	61 (13)	50 (14)	35 (7)	21 (4)	11 (2)
		rsp/min	1 (0.3)	1 (0.5)	1 (0.2)	1 (0.2)	1 (0.1)
R16	12	rft/hr	163 (71)	167 (44)	22 (11)	16 (7)	10 (4)
		rsp/min	6 (3)	9 (2)	1 (1)	1 (1)	1 (0.3)
	6	rft/hr	38 (29)	10 (8)	13 (9)	7 (3)	7 (4)
		rsp/min	1 (1)	0.2 (0.2)	0.3 (0.2)	0.3 (0.2)	0.3 (0.2)
R19	12	rft/hr	496 (51)	263 (35)	75 (10)	35 (7)	16 (4)
		rsp/min	30 (4)	26 (5)	8 (2)	4 (1)	2 (0.4)
	6	rft/hr	70 (21)	30 (10)	9 (5)	8 (2)	4 (2)
		rsp/min	1 (0.5)	1 (0.3)	0.2 (0.1)	0.3 (0.1)	0.2 (0.1)